CHECKING THE LOAD CAPACITY OF REINFORCED CONCRETE CORBEL THROUGH ELASTIC-LINEAR NUMERICAL ANALYSES WITH FINITE ELEMENT MODELING

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ABSTRACT

Short and very short corbels are commonly used as beam-column connection of precast, metallic, or composite concrete structures. Since the assumptions of beam theory does not apply in some of their regions, their analysis and design need to be meticulously performed, requiring the use of a strut-and-tie model or a friction-shear model. Due to the continuous technological advancement, it is possible to describe the behavior of these elements through structural analysis software. Therefore, the present study aims to compare the results of the computational analysis of one of the corbels tested by Fattuhi (1994), modelled using the finite element method (FEM) in SAP 2000 software, with the analyses performed on the same element using a strut-and-tie method based on NBR 9062:2027 and a truss containing the compressed strut and tie in its spans, based on what is stated in NBR 6118:2023 regarding the stresses developed in corbels. The results obtained suggest that the modelled corbel exhibited similar behaviour to that found through standards, also indicating shear failure due to insufficient longitudinal reinforcement and lack of transverse reinforcement. Thus, it can be concluded that the FEM modelling satisfactorily simulated the behaviour of the reinforced concrete corbel, resulting in values that were satisfactorily close to those observed in reality.

KEYWORDS: *Corbels. Finite Elements. Struts. Ties. Fracture.*

I. INTRODUCTION

Structural designs must be safe, cost-effective, and quickly executable. Thus, simplified solutions are commonly employed. Reinforced concrete corbels are precast or prefabricated connection elements that transfer horizontal loads to vertical elements, where the distance from the load application to the support face is less than or equal to its effective height [1]. These elements require careful analysis of the forces acting on their structure since in some regions the assumptions of beam theory, which the maintenance of the cross-sectional geometry of the element during deformation, do not apply.

Reinforced concrete corbels are prismatic elements that serve as supports for other elements and allow load transfer from columns and beams [2]. According to NBR 6118 [3], the design distinguishes between short and very short corbels based on their a/d ratio (figure below). The corbel is considered short if $0.5d \leq a \leq d$ and very short if $a < 0.5d$, where a is the distance from the applied load to the support face, and d is the effective height of the corbel. Figure 1 presents the necessary dimensions that a corbel must follow.

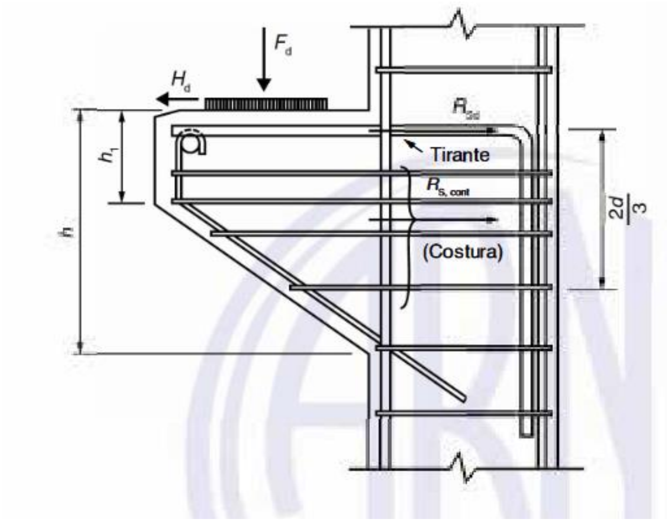

Figure 1 - Dimensions used in corbel classification.
Source: NBR 6118 [3].

The strut-and-tie (ST) or friction-shear models are relevant for corbel's design due to its geometric limitations and tendency to fail through shear. The Brazilian standard 6118 [3], as well as European, Canadian and American [4, 5, 6] acknowledge the STM as the main design method, while Chinese standard uses the truss model [7]. Other STM adaptations such as Softened Strut-and-Tie Method (SSTM) [8] and Russo Strut-and-Tie Method (RSTM) [9] are presented and evaluated in the work of [10]. This research, however, will focus on the STM and truss model.

NBR 6118 [3] does not allow the use of designs without shear reinforcement for short or very short corbels. Hence, it is of paramount importance to ensure a more ductile failure of the corbel to avoid load capacity loss. Short corbels must have shear reinforcement equal to 40% of the tie reinforcement, distributed in the form of horizontal stirrups equal to 40% of the tie reinforcement, and additionally, distributed in the form of horizontal stirrups at a height equal to 2/3 d.

Recently, structural analysis programs that evaluate internal stresses propagation within the corbels through finite element method (FEM) have been used as an alternative to conventional methods. This numerical approach divides the domain in several smaller elements and obtain a solution through solving differential equations [11]. Despite not being a new idea in the mathematical field [12] FEM software became an increasingly viable alternative considering the exponential advancement of computer processing capabilities over time.

Therefore, this work aims to model a reinforced concrete corbel using FEM in SAP 2000 software, and then analyze the same model through a strut-and-tie model and a truss model, comparing the results obtained through the procedures, aiming to confirm the efficiency and practicality of finite element modeling for the design of reinforced concrete corbels.

This paper is organized as follows. The corbel geometry and the standards considered for further analysis are defined in Section II. Section III presents results related to design methods elaborated in the previous section. Section IV exhibits discussion around the results obtained and, finally, Section V presents conclusions considering what was addressed throughout the sections presented.

II. METHODOLOGY

In this study, three analyses were conducted, each addressing the same problem in a different way. In the first stage, a short corbel from [13] work, which extensively tested a series of reinforced concrete corbels, was modeled using FEM with the structural analysis software SAP 2000 version 20. Next, the same element was designed according to NBR 9062 [14] using the strut-and-tie method (STM) as described by Fernandes and Debs [15]. Finally, the same corbel was analysed following NBR 6118 [3] specifications, calculating internal stresses with a truss model (TM). The methodology is resumed in the figure below.

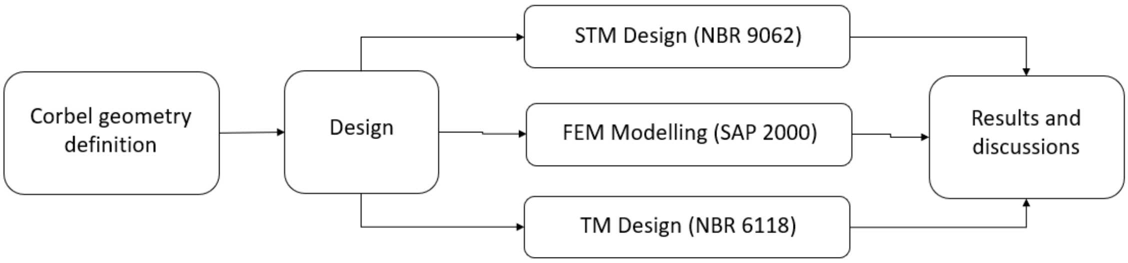

Figure 2 – Methodology resume.

2.1. Geometry considered

The selected model was taken from [13], who tested 38 reinforced concrete corbels of 150 mm x 150 mm x 200 mm with different effective heights under vertical loading, as shown in the figure below.

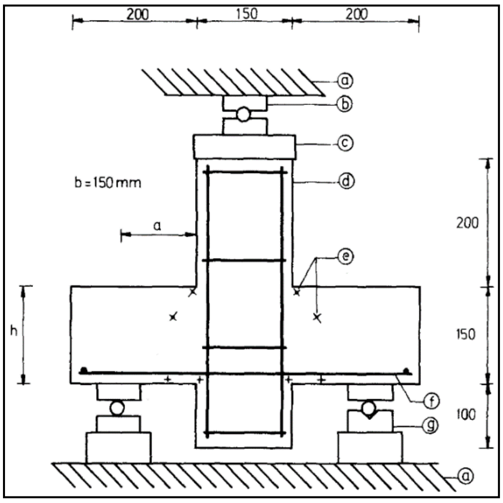

Figure 3 – Design of the tested corbel.
Source: [13].

Where a is the movable head of the testing machine, b is the spherical support, c is the loading plate, d is the concrete element, e is the strain gauge, f is the main reinforcement, g is the cylindrical support and h is the height of the corbel. Once the corbel was selected, it was modelled using FEM and beam and column elements, both utilizing the structural analysis software *SAP 2000 v. 20*.

Corbel number 73 from [13] study was chosen due to its simplified modeling factors, such as its geometry, the absence of metallic fiber reinforcement present in various elements of the study, and the location of the supports that simulate the load application points (75 mm from two opposite faces of the column).

Moreover, it was crucial to adopt a height for each finite element that would allow the closest approximation of the cross-sectional areas of the finite elements, simulating the reinforcements with the steel bar diameters used in the corbel tested by [13]. The geometry of corbel 73 is shown in the table below.

Table 1 - Characteristics of the studied corbel

CORBEL	MAIN REINFORCEMENT	a (mm)	b (mm)	d (mm)	h (mm)
73	2 x 8 mm	75	153.9	124	148

Source: adapted from [13].

Where a is the distance from the load application point to the face of the column, b is the width of the corbel's cross-section, d is the effective height of the corbel and h is the total height of the corbel. The chosen corbel is short (a/d = 0.6). Furthermore, all the elements tested in [3] were constructed, according to the author, with ordinary Portland Cement and potable tap water, and the column segments were built with 12 mm diameter longitudinal reinforcement and four 6 mm diameter stirrups. For corbel 73, modelled in this study, steel with f_y 451 MPa and fy 666 MPa was used.

III. RESULTS

3.1. Corbel structural analysis in SAP 2000

The feedback obtained from the software revealed a discretization into 62,816 finite elements, totalling a volume of 19,404 cm². The analysis provided, as one of the results, the deformed shape of the corbel and visually depicted the displacements and the paths of internal loads within the element, allowing the identification of the compressed struts in the corbel, as shown in the figures below:

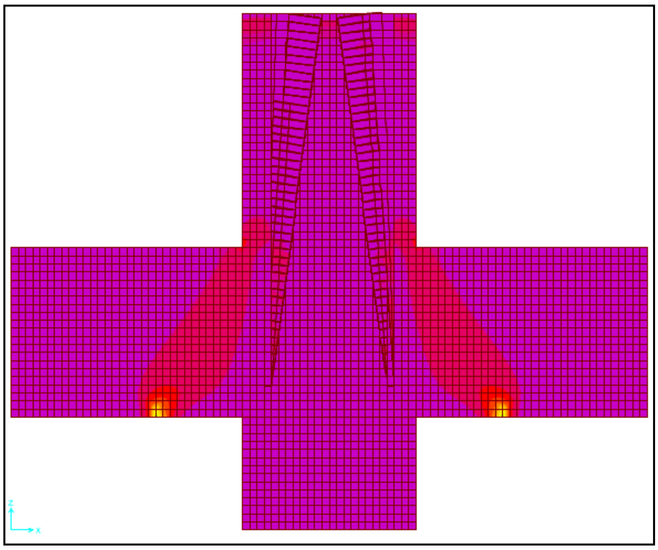

Figure 4 - Compressed struts in the corbel.
Source: SAP 2000

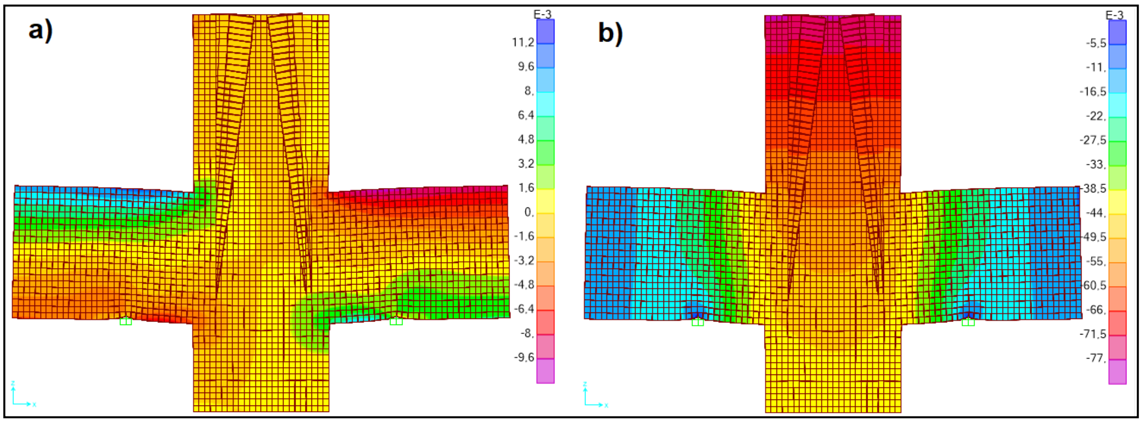

Figure 5 - Deformation: (a) horizontal; (b) vertical, mm.
Source: SAP 2000.

In Figure 5 (a), the displacements of the finite elements due to tensile and compressive stresses are represented, with cool colors indicating tension and warm colors indicating compression. Although the left side of the representation in Figure 5 (a) has the deformation values inverted, the position of the neutral axis (region without horizontal deformation) remains the same distance from the compressed face as that represented on the left side, which shows the deformations due to tension and compression without value inversion.

Figure 5 (b) shows the deformations in the y direction, with cool colors indicating the lowest values and warm colors indicating the highest values. By interpreting Figure 5, the largest displacements can be observed at the point of load application in the test (the upper part of the column) and smaller deformations at the ends of the corbels, which are the least stressed regions of the structure.

Additionally, it was possible to obtain the maximum and minimum forces acting on the corbel, as shown in the figure and table below:

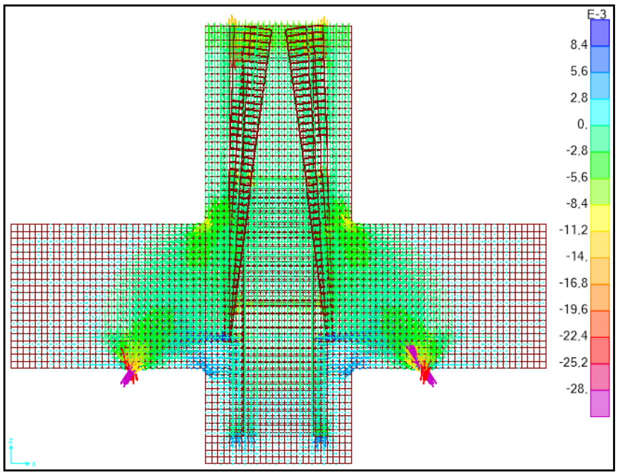

Figure 5 - Forces, kN/mm.
Source: SAP 2000.

Table 2 - Maximum and minimum values of forces, kN/mm²

RESULT	S11	S22	S33	S12	S13	S23	SMAX	SMID	SMIN	SVM
MAX	0.026	0.012	0.014	0.005	0.02	0.01	0.027	0.013	0.006	0.051
MIN	-0.025	-0.012	-0.049	-0.005	-0.02	-0.008	-0.012	-0.017	-0.061	1.00E-05

Through a cut made close to the beam-column intersection, it was possible to obtain approximate values of forces and moments in the x, y and z directions, as shown in the table below.

Table 3 - Forces and moments at the section near the beam-column junction

DIRECTION	FORCE (kN)	MOMENT (kN.m)
1	14.1	1.58
2	0.33	1.14
Z	30.15	0.78

Finally, the program provided the global values of the reactions in the structure, namely forces in the x, y, and z directions, and moments around the x, y, and z axes, as shown in the table below.

Table 4 - Base reactions.

GLOBAL Fx (kN)	GLOBAL Fy (kN)	GLOBAL Fz (kN)	GLOBAL Mx (kN.m)	GLOBAL My (kN.m)	GLOBAL Mz (kN.m)
-1.28E-08	3.58E-07	88.09	6.78	-24.22	1.02E-07

Source: Adapted from SAP 2000.

3.2. Design of the Corbel According to NBR 9062:2017

After performing the FEM of corbel 73, the design of the same element based on the NBR 9062 standard was initiated. For this purpose, a strut-and-tie model adapted from [4] was used, following the procedure outlined below:

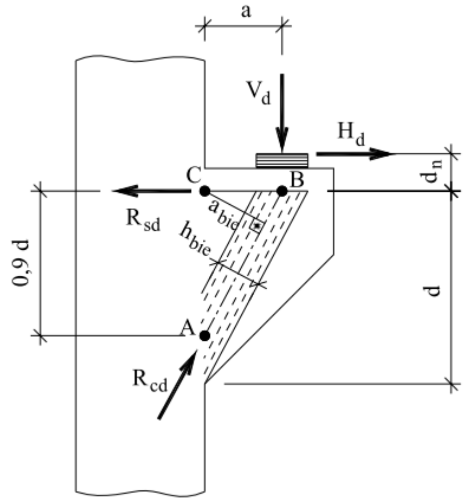

Figure 6 - Analysis model and geometric characteristics of short corbel
Source: [5], adapted from [4].

Where: V_d is the vertical load applied directly on the upper face of the corbel, H_d is the horizontal load applied on the side face of the corbel, d_h is the vertical distance between the point of load application and the tie, d is the effective height of the corbel, a is horizontal distance between the point of load application and the face of the pillar. The perpendicular distance between the node of the beam-column intersection and the axis of the compressed (a_{bie}) strut is given by:

$$a_{bie} = \frac{0{,}9da}{\sqrt{0{,}9^2 + a^2}} \tag{2-1}$$

The height of the compressed strut (h_{bie}) is given by:

$$h_{bie} = 0{,}2d \tag{2-2}$$

From the moment equilibrium with respect to point A and considering d_h/d close to 0.2, it is possible to obtain the necessary reinforcement for the tie. (A_{stir}) is given by:

$$A_{stir} = \frac{V_d}{f_{yd}}\frac{a}{0{,}9d} + \frac{H_d}{f_{yd}} \tag{2-3}$$

From the moment equilibrium with respect to point C:

$$R_{cd} = \frac{V_d a + H_d d_h}{a_{bie}} \tag{2-4}$$

The compression stress in the strut is obtained. (σ_{cd}) is given by:

$$\sigma_{cd} = \frac{R_{cd}}{0{,}2bd} \tag{2-5}$$

Substituting (2-4) into (2-5) and considering the horizontal load (H_d) negligible, we have:

$$\sigma_{cd} = \frac{V_d}{0{,}18bd}\sqrt{0{,}9^2 + \left(\frac{a}{d}\right)^2} \tag{2-6}$$

The equation (2-6) is upper-bounded by fcd, in cases of direct forces, or by $0{,}85fcd$, in cases of indirect forces.

Similarly, equation (2-6) expressed in the form of reference tangential stress yields:

$$\tau_{wd} = \frac{V_d}{bd} \leq \tau_{wu} \tag{2-7}$$

where:

$$\tau_{wu} = \frac{0{,}18\beta f_{cd}}{\sqrt{0{,}9^2 + \left(\frac{a}{d}\right)^2}} \tag{2-8}$$

For the design, the following considerations were made: d=0,124m, a=0,075m, b=0,154m, f_{ck}=25MPa, f_{cd}=17,86MPa, f_y=451MPa e f_{yd}= 392,17MPa. The table below shows the results of the design:

Table 6 - Design result via MBT.

a_{bie} **(cm)**	h_{bie} **c(m)**	V_d **(kN)**	$A_{s_{tir}}$ **(cm²)**	R_{cd} **(kN)**	σ_{cd} **(MPa)**	τ_{wd} **(MPa)**	τ_{wu} **(MPa)**
6.22	2.48	43.99	0.75	35.7	13.88	2.3	2.96

3.3. Design of the Corbel as a Truss

According to NBR 6118 [3], the behavior of short corbels can be described through a strut-and-tie model that considers the global equilibrium of the structure. Furthermore, the standard states that the tie anchors to the pillar on one side and to the strut under the vertical load on the other side. Additionally, the compressed strut utilizes the entire available height of the corbel, extending from the point of load application to the face of the pillar or support.

Therefore, knowing the heights of the strut, calculated from equation (2-2) and 0.9d (distance between the anchoring points of the tie and strut on the pillar), as shown in Figure 6, a simple truss was developed to determine the internal tensile and compressive forces acting on the tie and strut of the corbel studied in this work, generated from the applied load, as shown in the figure below:

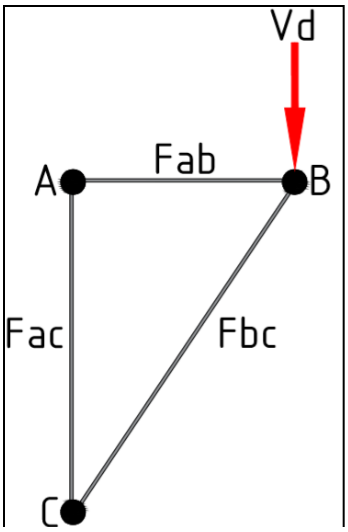

Figure 7 - Truss representing the tie and strut inside the corbel.

In the truss of Figure 7, the segment Fab represents the distance between the point of load application and the face of the pillar (75mm), and Fac, which is perpendicular to Fab, is equal to 0.9d (111.6mm). Therefore, by the Pythagorean theorem, Fbc is equal to 134.46mm, and the angles ABC and ACB are equal to arctan (Fac/Fab) and arctan (Fab/Fac), resulting in 56.09° and 33.90°, respectively. These angles are within the range permitted by NBR 6118 [3], which states that inclined struts must have a tangent inclination between 0.57 and 2 (29.68° to 63.43°).

With this information, the stresses in the truss segments were calculated from the equilibrium equations at nodes A, B, and C. The result is shown in the table below:

Table 7 - Forces acting on the tie and strut.

FORCE	VALUE (kN)	NATURE
FAB	29.56	TENSION
FAC	43.99	COMPRESSION
FBC	53	COMPRESSION

From this point, the stress equation was used to obtain the compressive stress of the compressed strut of the element. The segment Fbc, which represents the compressed strut of the corbel, is subjected to a compressive force of 53 kN, acting over a cross-sectional area with a length of (0.024m) and a width of b (0.154m), resulting in a compressive stress of 13.88 MPa. To obtain the required steel area in the tie Fab, according to NBR 6118 [3], the equation was used:

$$A_s = \frac{f_{sd}}{f_{yd}}$$

with F_{Sd}: 29.56 kN and f_{yd} 45.1 kN/cm² for A_S: 0.75 cm². The tensile stress in tie F_{ab}, calculated with F: 29.56 kN and AAA: 0.75 cm² resulted in 3.94 MPa.

IV. DISCUSSIONS

The NBR 9062 [14] determines the use of shear reinforcement in corbels with a/d ratio ≤ 1, concentrated at 2/3d, with a minimum value, in cm² per meter, of 0.15b. However, NBR 6118 [3], considers it in the design of these elements. The corbel studied in this research had only longitudinal reinforcement and, subjected to a vertical load of 87.5 kN by [3], exhibited shear failure.

The specimen was modeled in SAP 2000 software, considering it to be made of C25 concrete, since [3] executed corbel 73 in ordinary Portland cement with a ratio of 1:3:3 and a water-cement ratio of 0.77. The analysis resulted in a maximum compression stress of 25 MPa in the compressed strut, a region of discontinuity just below the point of load application of the supports simulating the load, where it was possible, through the discretization of the element, to obtain the value of the acting stress. The maximum tensile stress obtained was 26 MPa, near the junction of the face where the supports were applied with the pillar. The mentioned values are in Table 3, direction S11.

Through the design by STT, it was possible to measure the steel area required in the cross section of the element to support the load of 87.5 kN added to the self-weight of the element (Table 4), which is 25% lower than that used during the test (1.0 cm²), which rules out the hypothesis of failure due to excessive yielding of the longitudinal reinforcement, as the reinforcement was able to withstand the tensile stresses developed in the corbel.

Additionally, the distance from the point of application of the supports simulating the load during the test to the end of the corbel (125mm) alone rules out the hypothesis of failure due to anchorage failure, which, according to [16,17], occurs in cases where the distance between the applied force and the end of the corbel is small, which was not the case.

The stresses in the truss (Table 7) corroborated with those obtained through the analysis by the strut-and-tie method. The segment F_bc, which represents the compressed strut of the corbel, resulted in the same calculated value for σ (13.88 MPa). For both analyses, the compressive stress was lower than the design strength of reinforced concrete, defined by NBR 6118 [3] as fcd = Fck/1.4 (17.86 MPa), ruling out the hypothesis of failure due to crushing of the compressed strut. The tensile stress obtained for the tie through F_ab (3.94 MPa) corroborates the conclusion that there was no failure due to flexural failure due to excessive yielding of the longitudinal reinforcement.

Through finite element modeling, shear stresses were obtained for the load of 87.5 kN added to the specimen's self-weight, acting in various regions of the corbel, spreading throughout the structure from the supports to the pillar. Additionally, it was possible to observe the values of these stresses, between 5 MPa and 10 MPa (values taken from Table 3, specifically in S12, S13, and S23). It is worth noting that all stresses in the range between 5 MPa and 10 MPa are well above the shear resistance obtained by equation (2-7), equal to 2.3 MPa, pointing to shear failure as the most likely hypothesis, according to the FEM modeling.

V. CONCLUSIONS

In the computational model, it was possible to observe the propagation of stresses, their values, and the displacements after the deformation of the element. Additionally, the results of the computational simulation adequately represented the real behavior of the reinforced concrete corbel, even allowing to predict the mode of failure (shear) that occurred in the tested specimen, through the wide range of results presented after the modeling.

Regarding normative recommendations, it was possible to observe the inaccuracy of the results regarding the tested corbel, since the standards do not consider shear reinforcement for the calculation of resistances.

Therefore, it was concluded that the design of the reinforced concrete corbel according to normative criteria did not guarantee the satisfactory performance of the specimen, as the design did not ensure the safety of the structure.

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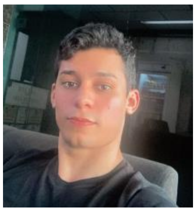

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