SPARSE PLANT IDENTIFICATION USING THE IPNLMS ALGORITHM WITH PARAMETER OPTIMIZATION VIA GRAVITATIONAL SEARCH

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ABSTRACT

Problems of identifying systems involving sparse plants have been objects of study, mainly regarding the use of adaptive filters. In this work, the behaviour of the IPNLMS (Improved Proportionate Normalized Least-Mean-Square) filter in estimating the coefficients of a sparse plant is investigated. This algorithm has, in essence, several parameters. To optimize the performance of IPNLMS, a gravitational search algorithm is used to estimate the optimal values of the proportionality parameter inherent to the individual gain of each coefficient. The results obtained indicate that the proposed methodology, when compared to other algorithms, presents a better convergence speed, but converges to a significantly higher error.

KEYWORDS: *Systems Identification, Adaptive Filter, Sparse Plant, Gravitational Search, IPNLMS.*

I. INTRODUCTION

According to [1] problems such as estimation of harmonic components in electrical power systems, echo cancellation in telecommunications and identification of seismic events are applications that have a high degree of sparsity. A sparse vector or matrix has a small part of its non-zero elements and the rest have null values or very close to zero [2].

Identification of Systems whose plan has a high degree of sparsity is, above all, an important problem to be investigated not only due to the different applicability but also due to the mathematical and computational challenges that the problem presents [3].

To always improve the identification of systems, several algorithms have been developed and improved. Among these algorithms are LMS (Least Mean Square), NLMS (normalized LMS), PNLMS (Proportional) and IPNLMS (Improved Proportionate Normalized Least-Mean-Square). All of these algorithms have their performance dependent on parameters inherent to the mathematical formulations [4].

The parameters of adaptive filters range from learning rate (or step) to those that imply significant changes in some process variables, as is the case with the proportionality parameter inherent to the individual gain function of the IPNLMS filter coefficients. This parameter, in turn, directly influences the individual gain and, consequently, the convergence of the [5] filter.

Proposing computational models that can estimate the best proportionality parameter for the individual gain of the IPNLMS filter when it is used to identify sparse plants was one of the objectives of [6] work. In this work, the author took an approach using the Tabu and Golden Ratio search algorithms.

In this context, this work proposes an analysis of the "behaviour" of the IPNLMS filter with the optimization of the proportionality parameter through the Gravitational Search algorithm (GSA) [7].

Monte Carlo simulations are carried out to evaluate the algorithm. Comparisons between the proposed method and the LMS, NLMS, PNLMS filters are presented. These comparisons are made considering the graphical behaviour of the misalignment measure. The way in which the non-zero coefficients converge to the real values is also evaluated graphically.

II. ADAPTIVE FILTERING

According to [8], adaptive filters are an important part of digital signal processing, especially when the study and/or application environment is statistically unknown. Among the large number of adaptive filtering algorithms LMS (Least-Mean-Square) is the most basic and most used algorithm [9].

The LMS weights adaptation rule is based on the Stochastic Gradient method and is given by the following relation [10]:

$$w(n+1) = w(n) + \mu\, e(n)\, x(n) \tag{1}$$

where $w(n+1)$ is the coefficient in step $n+1$, $w(n)$ is the coefficient in step n, μ is the value of the adaptation step (or rate learning process), $x(n)$ is the input signal and $e(n)$ is the difference between the filter output and the desired signal, that is, the estimation error. Below is the adaptive algorithm known as LMS (Table 1).

Table 1. Algorithm LMS

Algorithm 1 LMS

1: Initialization and parameters

$$\mathbf{w}(0) = \mathbf{0}$$
$$0 < \mu < \frac{1}{\lambda_{\max}}$$

2: Input and output data from the plant and the adaptive filter

$$d(n) = \mathbf{x}^T(n)\mathbf{p}(n)$$
$$y(n) = \mathbf{x}^T(n)\mathbf{w}^T(n)$$

3. Calculation of the error sign

$$e(n) = d(n) - y(n) + v(n)$$

4: Update filter coefficients

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n)\mathbf{x}(n)$$

Several filters have been obtained from changes in the LMS. Among these filters, NLMS (Normalized LMS), PNLMS (Proportional Normalized Algorithm) and IPNLMS (Improved) stand out. The difference, in terms of implementation, between LMS and IPNLMS is that the latter computes an individual gain of the filter coefficients [11]:

$$g_i(n) = (1-\alpha)\frac{1}{2N} + (1+\alpha)\frac{|w_i(n)|}{2\|\mathbf{w}(n)\|_1 + \varsigma}$$

where $\varsigma > 0$ is a regularization parameter used to avoid division by zero. The factor α is called the proportionality parameter.

The rule for updating the coefficients of the IPNLMS algorithm is given by:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \frac{\mu \boldsymbol{G}(n) e(n) \boldsymbol{x}(n)}{\boldsymbol{x}^T(n) \boldsymbol{G}(n) \boldsymbol{x}(n) + \varepsilon}$$

where μ is the adaptation step and $\varepsilon > 0$ is a numerical regularization parameter to stabilize the solution. The matrix

$$\mathbf{G}(n) = \text{diag}\,[g_1(n) \quad g_2(n) \quad \dots \quad g_N(n)],$$

of order N x N is responsible for the distribution of individual gains $g_{\text{i}}(n)$, controlling the adjustment of the ith coefficient of the algorithm. The diag operator defines a diagonal matrix whose elements $g_1(n) \quad g_2(n) \quad \dots \quad g_N(n)$ make up the main diagonal. The sequence of steps of the IPNLMS algorithm is described below (Table 2) [11].

Table 2. Algorithm IPNLMS

Algorithm 2 IPNLMS

1: Initialization and parameters

$$\mathbf{w}\,(0) = \mathbf{0}$$

$$0 < \mu < 2$$

$$\varepsilon > 0$$

$$-1 \leq \alpha < 1$$

$$\varsigma > 0$$

2: Plant input and output data

$$d(n) = \mathbf{x}^T(n)\mathbf{p}(n)$$

and the adaptive filter

$$y(n) = \mathbf{x}^T(n)\mathbf{w}(n)$$

3. Error signal

$$e(n) = d(n) - y(n) + \upsilon(n)$$

4: Individual gain of adaptive filter coefficients

$$g_i(n) = (1-\alpha)\frac{\mathbf{1}}{\mathbf{2N}} + (1+\alpha)\frac{|w_i(n)|}{2||\mathbf{w}(n)||_1 + \varsigma}, \qquad i = 1,2,3,\dots,N$$

5: Individual earnings matrix (N x N)

$$\mathbf{G}(n) = diag[g_1(n) g_2(n) \cdots\ g_N(n)]$$

6: Update adaptive filter coefficients

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu \mathbf{G}(n) e(n) \mathbf{x}(n)}{\mathbf{x}^T(n)\mathbf{G}(n)\mathbf{x}(n) + \varepsilon}$$

III. GRAVITATIONAL SEARCH ALGORITHM

The Gravitational Search Algorithm — GSA is a global optimization algorithm based on the laws of gravity and Newtonian dynamics. In this algorithm, candidate solutions are represented by particles, which attract each other, in accordance with the Law of Gravity; and they move, according to the laws of dynamics [12].

In GSA, each candidate solution is represented by a particle and each problem variable is represented by a position coordinate, according to the following equation:

$$\boldsymbol{x_i}(\mathrm{t}) = (x_{\mathrm{i1}}(t), x_{\mathrm{i1}}(t), x_{\mathrm{i1}}(t), .. x_{\mathrm{iD}}(t)), \qquad i = 1, 2, 3, \dots, P.$$

where $\boldsymbol{x_i}(\mathrm{t})$ is the ith solution, that is, the position of the particle i in the iteration t, x_{ij}, the variable j of the solution i, P is the number of candidate solutions and D is the number of variables in the problem [7]. The sequence of steps for executing the GSA is listed below.

Step 1 - Definition of the search space
Step 2- Random initialization
Step 3- Update G(t)

$$G(t) = G(t_0) \,.\left(\frac{t_0}{t}\right)^{\beta}, \quad \beta < 1,$$

where G(t) is the value of the gravitational constant at time t. G(t_0) is the value of the gravitational constant in the first cosmic quantum interval of time t_0.
Step 4 - Calculation of the total force in different directions.

$$F_t^d(t) = \sum_{j=1, j\neq i}^{N} rand_j F_{ij}^d(t),$$

where $rand_j$ is a random number in the range [0,1] e $F_{ij}^d(t)$ the force acting on the mass "i" from the dough "j" is given by
Step 5 - Calculation of acceleration a_i^d and velocity v_i^d
Step 6 - Update the agents' position

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1)$$

Step 7- Repeat steps 3 to 6 until the stopping criterion is reached.
Step 8 - End

The figure below shows the sequence of steps of the GSA algorithm (Figure 1).

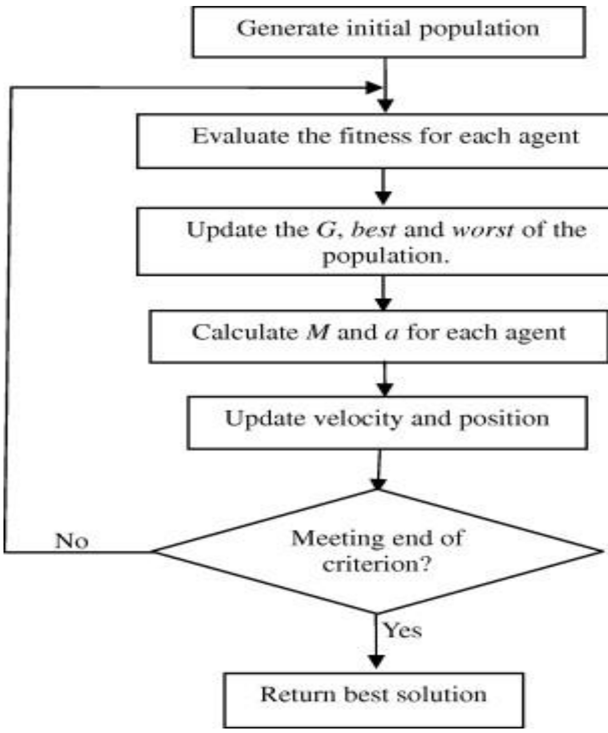

Figure 1. GSA Algorithm Flowchart [12]

The main application of GSA is to find critical points (maximum and minimum) of functions. In this context then, this algorithm will be used in this work with the purpose of finding the proportionality α parameter of the IPNLMS algorithm.

IV. MATERIALS AND METHOD

The proposed methodology consists of analysing the IPNLMS optimizing the proportionality parameter. In this way, an optimization problem is introduced to the IPNLMS algorithm in which an objective function is defined, and a parameter analysed in the minimization process. The optimization problem to be solved then consists of the following formulation:

$$\alpha^* = \arg\ \min \varphi(n)$$

considering a search space $-1 < \alpha < 1$.

The objective function to be minimized is known as a posteriori quadratic error [2].

$$\varphi(n) = 10\ \log[d(n) - x^T(n)w(n+1) + z(n)]^2$$

It is important to note that $\varphi(n)$ depends indirectly on α. What really depends on the α parameter are the individual gains of each filter. However, updating the weights depends on the gain matrix and as $\varphi(n)$ depends on the weights, this depends on α.

Another important observation is that according to the proposed optimization problem, the α parameter optimizes the error a posteriori, that is, the function to be minimized is the possible filter output error. This error is normalized by the logarithmic function, meaning that it is given in dB. The following algorithm summarizes the steps of the proposed methodology to optimize the IPNLMS proportionality parameter (Table 3).

Table 3. Algorithm IPNLMS-GSA

Algorithm 3 IPNLMS-GSA

1: Initialization and parameters

$$\mathbf{w}\,(0)\ =\ \mathbf{0}$$

$$0 < \mu < \ 2$$

$$\varepsilon > 0$$

$$-1 \leq \alpha < 1$$

$$\varsigma\ > 0$$

2: Plant input and output data

$$d(n) =\ \mathbf{x}^T(n)\mathbf{p}(n)$$

and the adaptive filter

$$y(n)\ =\ \mathbf{x}^T(n)\mathbf{w}(n)$$

3. Error signal

$$e(n) = d(n) - y(n) + v(n)$$

4: Using the GSA algorithm to calculate the optimal proportionality parameter

5: Individual gain of adaptive filter coefficients

$$g_i(n) = (1-\alpha)\frac{\mathbf{1}}{\mathbf{2N}} + (1+\alpha)\frac{|w_i(n)|}{2||\mathbf{w}(n)||_1 + \varsigma}, \qquad i = 1,2,3, \dots, N$$

6: Individual earnings matrix (N x N)

$$\mathbf{G}(n) = diag[g_1(n) g_2(n) \cdots \ g_N(n)]$$

7: Update adaptive filter coefficients

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu \mathbf{G}(n) e(n) \mathbf{x}(n)}{\mathbf{x}^T(n) \mathbf{G}(n) \mathbf{x}(n) + \varepsilon}$$

The difference between IPNLMS and IPNLMS-GSA is step 4, i.e., parameter optimization α.

The filter analysis proposal considers a sparse impulse response with N = 100 coefficients, with the values of its active coefficients equal to p = {0.1, 1.0, − 0.5, 0.1} located at positions {1, 30, 35, 85}, respectively.

The metric used to evaluate and compare the performance of algorithms is misalignment. Mathematically this metric is given by [2]:

$$\kappa(n) = 10 log \frac{\|p - w(n)\|_2}{\|p\|_2}$$

Below is an analysis of the solutions carried out considering the proposed methodology.

V. Results and Discussion

For comparison and analysis purposes, simulations were carried out comparing classical algorithms in the literature (LMs, NLMS, PNLMS) with IPNLMS-GSA (IPNLMS with parameter optimization through gravitational search). 100 Monte Carlo simulations were carried out. The step of the LMS considered was 0.001 and of the other algorithms 0.3. The parameters inherent to the NLMS and PNLMS filters were set at $\varepsilon = 0.001$ and $\rho = 0{,}5$. As an input signal, a correlated signal given by the autoregressive relationship was considered:

$$x(n) = 0{,}4\, x(n-1) + 0{,}4\, x(n-2) + v(n)$$

where $v(n)$ is white noise with variance 0.77.

The measurement noise added to the input signal is white with variance 10^{-3}. To evaluate the performance of the algorithm, normalized misalignment (in dB) is used. The Figure 1 below shows the results of the averages of the Monte Carlo simulations, comparing the classic algorithms from the literature and the IPNLMS-GSA.

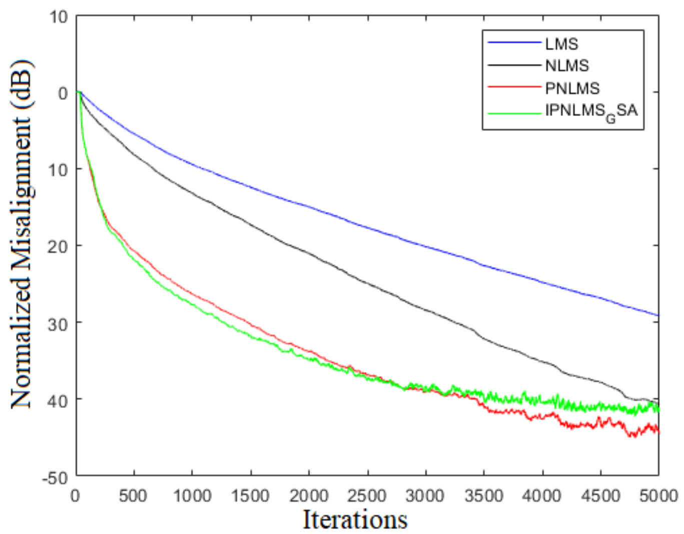

Figure 2. Misalignment (Plant “p”)

Observing Figure 1 the results of the PNLMS and IPNLMS-GSA algorithms are quite similar. However, IPNLMS – GSA converges to the permanent stage significantly faster and IPNLMS achieves the lowest error among the compared algorithms.

In view of the similarities between the results of the IPNLMS and IPNLMS-GSA filters, a comparison was carried out between these algorithms, considering the individual convergence of each non-zero coefficient of the sparse plant (Figure 2).

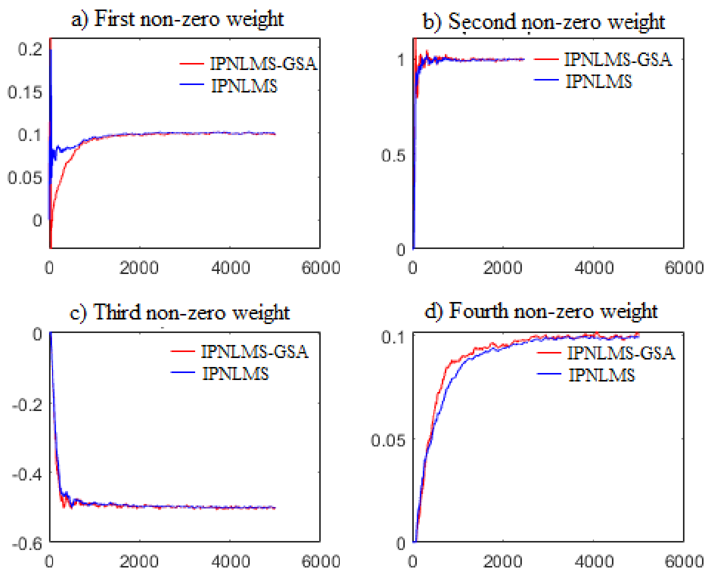

Figure 3. Non-zero coefficients (Plant "p")

The convergence of non-zero coefficients (Figure 2) demonstrate the similarity in the "behaviour" of the IPNLMS-GSA and PNLMS filters, as the instants in which the algorithms converge to the real coefficients are close. To collaborate with the proposed analysis, a change in the plant was considered. This change consists of using "-p". The Figure 3 graphically demonstrates the misalignment of the compared filters.

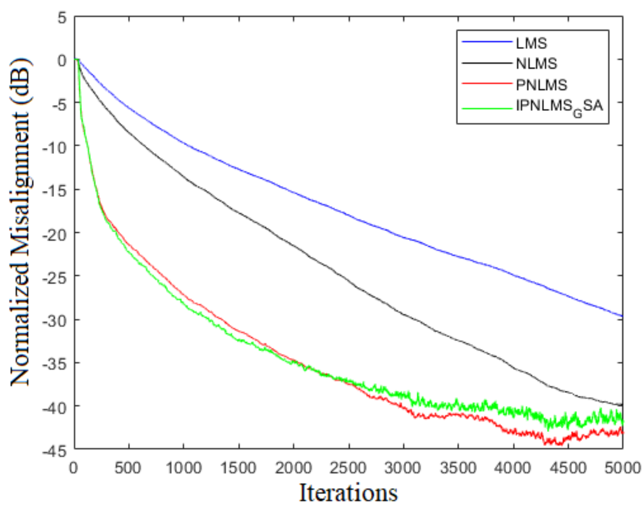

Figure 4. Misalignment (Plant "- p")

Even making the change in the plant, the misalignment results remain like the first simulation, that is, the IPNLMS-GSA enters steady state a little before the PNLMS, which in turn achieves the lowest error. The convergence of the coefficients of this plant were also analysed. The Figure 4 shows the graphs with the convergence performance of the non-zero coefficients obtained by PNLMS and IPNLMS-GSA.

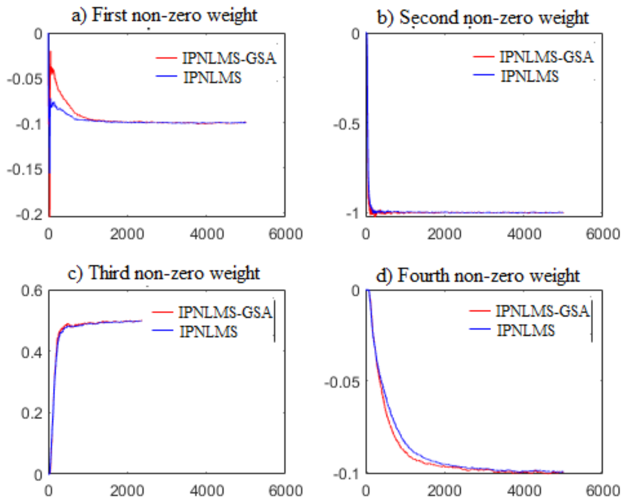

Figure 5. Non-zero coefficients (Plant "-p")

It is interesting to note that the convergences of the second and third coefficients are practically the same. On the other hand, the first coefficient converges "faster" when the PNLMS filter is used. The last non-zero coefficient converges when IPNLMS-GSA is used.

VI. CONCLUSIONS

After observing the simulations carried out, it is possible to highlight the great similarity (in general) between the results of the IPNLMS-GSA and PNLMS filters. However, it is worth highlighting that the use of the GSA algorithms allowed the application of the IPNLMS not to be dependent on the choice of the proportionality parameter.

In addition to the independence of the proportionality parameter, the IPNLMS algorithm presented a shorter steady-state entry time than the other algorithms, especially when compared to LMS and NLMS. In summary, the simulations indicated that it is possible to give up the pre-defined choice of the IPNLMS proportionality parameter to optimize your choice.

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