CHECKERBOARD PROBLEM IN FINITE ELEMENT BASED TOPOLOGY OPTIMIZATION

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ABSTRACT
It is well known that traditional formulations of topology optimization that make use of finite element method suffer from instabilities such as checkerboarding. This checkerboarding is generally due to mathematical instability, commonly observed solution of minimum compliance problems. If checkerboarding problem is controlled in an efficient way we can obtain more accurate and highly optimized results in Topology Optimization. This paper reviews the various available checkerboarding control methods available and presents a comparison of checkerboarding control methods. As a designer one must know which control method should be applied to get desired result also insight into the checkerboarding opens a new field for developing more able algorithms for controlling checkerboarding problem.

KEYWORDS: *Topology optimization, checkerboarding, finite element method.*

I. INTRODUCTION

Nowadays, there are commercial programs to solve only simple topology optimization problem. When developing a new computer code, many computational and theoretical issues appear. The most common are the following: a) checkerboard patterns; b) mesh dependency; c) local minima; and d) singular topologies (for stress constrained problems).

A checkerboard is defined as a periodic pattern of high and low values of Pseudo-densities, x_j arranged in a fashion of checkerboards. This behaviour is undesirable as it is the result of a numerical instability and does not correspond to an optimal distribution of material. The checkerboards possess artificially high stiffness, and also such a configuration would be difficult to manufacture. Checkerboard patterns are formed due to bad numerical modelling of the stiffness of the checkerboards

Figure 1: The checkerboard pattern in simply supported beam example [Sigmund and Petersson, 1998]

II. METHODS TO OVERCOME CHECKERBOARDING

To overcome the checkerboarding at least four types of methods are proposed: Checkerboarding is still a area of constant research to develop more and more efficient algorithms and method to improve Topology Optimization.

2.1 Use of high order finite element

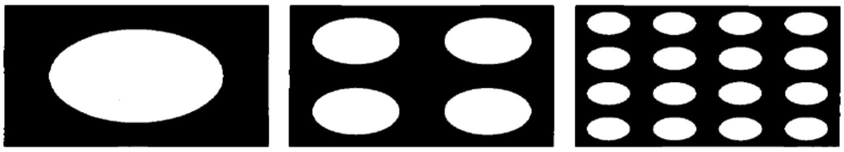
Sigmund and Petersson [1998] suggested the use of higher order finite elements for the modelling of the structure so that the stiffness properties of checkerboard patterns can be accurately calculated and checkerboards are avoided. Use of linear shape function finite elements for descritization of structures, gives rise to generation of checkerboard patterns. Checkerboards are typically prevented when using 8 or 9-node quadrilaterals for the displacements in combination with an element wise constant discretization of density. The use of higher order finite elements in topology design results in a substantial increase in CPU-time (even though this is not today a serious problem for 2-D problems). Eight node elements provides more accurate results for mixed (quadrilateral-triangular) automatic meshes and can tolerate irregular shapes without as much loss of accuracy. The 8-node elements have compatible displacement shapes and are well suited to model curved boundaries.

The 8-node element is defined by eight nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element may be used as a plane element or as an axisymmetric element. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities

(a) (b)

Figure 2: (a) The topology optimization result using four-node element, (b) The topology optimization result using eight-node element

2.2 Perimeter control technique



$V = 1 - 1.\pi\frac{1}{3^2} = 0.65$ $\quad$ $V = 1 - 4.\pi\frac{1}{6^2} = 0.65$ $\quad$ $V = 1 - 16\pi\frac{1}{12^2} = 0.65$

$P = 1.2\pi\frac{1}{3} = 2.09$ $\quad$ $P = 4.2\pi\frac{1}{6} = 4.19$ $\quad$ $P = 16.2\pi\frac{1}{12} = 8.38$

Figure 3: Smaller holes increase the perimeter, for a fixed volume. V is the volume and *P* is the perimeter of internal holes.

The perimeter of a mechanical element Ω^{mat} is, vaguely speaking, the sum of the lengths/areas of all inner and outer boundaries. Constraining the perimeter clearly limits the number of holes that can appear in the domain and existence of solutions to the perimeter controlled topology optimization is actually assured for both the discrete 0-1setting and the interpolated version using SIMP Also, it has been implemented for both situations and for 2-D as well as 3-D problems. For the SIMP method, one can impose a constraint that mimics such a perimeter bound in the form of an upper bound on the *total variation*, *TV (ρ)*, of the density *ρ*. In case the function *ρ* is smooth, the total variation constraint is a L^1 bound on its gradient

$$TV(\rho) = \int_{R^n} \|\nabla\rho\| dx \leq P^* \qquad 2.2.1$$

For a 0-1 design, the total variation of *ρ* coincides with the perimeter of Ω^{mat} when *ρ* is l *in* Ω^{mat} and 0 elsewhere. In this case the constraint is expressed as

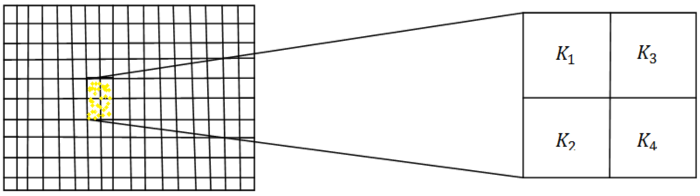
$$TV(\rho) = \sup\left\{\int_{R^N} \rho \operatorname{div}\varphi \, dx \mid \varphi \epsilon C_c^1 (R^n, R^n), \|\varphi\| \leq 1\right\} \leq P^* \qquad 2.2.2$$

Where $C_c^1(R^n, R^n)$ denotes compactly supported vector valued C^1 functions. For an element wise constant finite element discretization of the density the total variation can in 2-D be calculated as

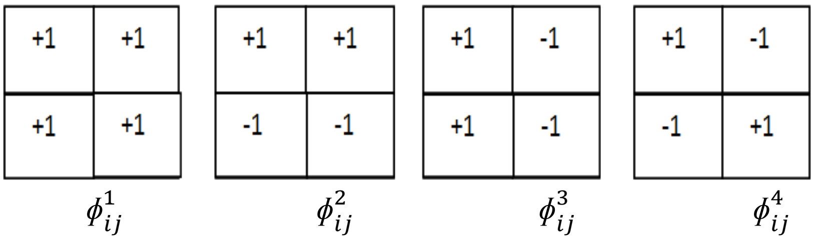
$$P = \sum_{k=1}^{K} l_k \left(\sqrt{\langle\rho\rangle_k^2 + \epsilon^2} - \epsilon\right) \quad 2.2.3$$

Where $\langle\rho\rangle_k$ is the jump of material density through element interface k of length l_k and K is the number of element interfaces (here one should also count interfaces at the boundary of the domain Ω — else there will be bias towards having material at the borders of Ω). The parameter ϵ is a small positive number which is used to convert the non-differentiable absolute value into a differentiable term. This expression is exactly the total variation of the element-wise constant density when ϵ = 0.

2.3 Patch technique



Ω (a) Patch of four elements P_{ij}, (2M columns and 2N rows)



(b) Basic functions associated with P_{ij}

Figure 4: Patches and basis functions used for checkerboard control.

In order to save CPU-time, but still obtain checkerboard free designs, it has been suggested to employ a patch technique. This technique has in practical tests shown an ability to damp the appearance of checkerboards. The strategy controls the formation of checkerboards in meshes of 4-node quadrilateral displacement elements coupled with constant material properties within each element. Thus one maintains the use of low order elements. However, the end result is the introduction of some type of element with a higher number of nodes, as the method in effect results in a "super-element" for the density and displacement functions in 4 neighboring elements. In what follows we will assume that the design domain ft is rectangular. It is discretized using a uniform mesh of square, 4-node iso-parametric element $K_{ij}, i = 1, \ldots, 2M, j = 1 \ldots, 2N$ where 2M and 2N are the (even) number of elements per side. Consider now, for odd i and j, a patch P_{ij} of four contiguous elements $K_1 = K_{i,j}, K_2 = K_{i+1}, K_3 = K_{i,j+1}$ and$K_4 = K_{i+1,j+1}$, as shown in Fig. 4, i.e,

$$P_{ij} = K_1 U K_2 U K_3 U K_4. \quad 2.3.1$$

Associated with P_{ij} we introduce basis functions $\phi_{ij}^1, \phi_{ij}^2, \phi_{ij}^3$ *and* ϕ_{ij}^4 which take the values +1 in P_{ij} according to the pattern shown in Fig. 4 and are zero outsideP_{ij}. Here we note that:

— The functions $\{\phi_{ij}^k\}$ constitute an orthogonal basis,

A "pure" checkerboard pattern is of the form $u = \sum_{P_{ij}} u_{ij}\phi_{ij}^4$ suggests that in order to avoid the formation of checkerboard patterns we need to restrict ρ to lie within the more restricted, checkerboard-free space

$$\tilde{V} = \left\{v \middle| v(x) = \sum_{P_{ij}} \left(v_{ij}^1\phi_{ij}^1 + v_{ij}^2\phi_{ij}^2 + v_{ij}^3\phi_{ij}^3\right), \left(v_{ij}^1, v_{ij}^2, v_{ij}^3\right) \epsilon R^3\right\} \quad 2.3.2$$

Where $i = 1,3, \ldots 2N - 1, j = 1,3, \ldots 2M - 1$

This restriction on ρ links the four elements in a patch, and the amount of material in $K_1 U K_4$ equals that of $K_2 U K_3$ and each is half of the total volume of the patch.

2.4 The Poulsen scheme

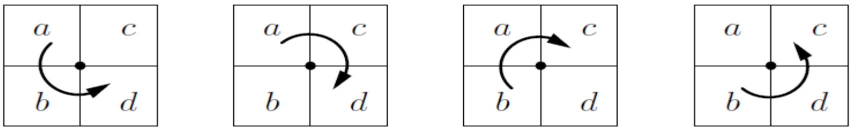


Figure 5: A check for monotonicity along four paths around an interior node.

A simple scheme to prevent checkerboard pattern and one-node connected hinges in topology optimization, was proposed by Poulsen. The scheme is particularly important to overcome one-node connected hinge that are often seen in topology optimization of compliant mechanism, since checkerboarding effects can of course be removed through the use of normal filters. By a one-node connected hinge it is understood that four elements surround a node, and only two opposing elements are filled with material, while the other two opposing elements are empty. These one-node connected hinges are of course also the blocks of checkerboard pattern. Here we define a non negative constraint function that should have value zero for the design to be free of checkerboard.

Consider the patch of square elements in Fig. 5. Defining the function

$$m(x,y,z) = |y - x| + |z - y| + |z - x| \quad 2.4.1$$

that is zero if the sequence of real numbers *x, y, z* is monotonic (increasing, decreasing or constant) and strictly positive otherwise, we can determine that the patch is free of checkerboard patterns, if just one of the numbers *m(a, b, d), m(a, c, d), m(b, a, c)* or *m(b, d, c)* is zero. This can be in turn be expressed as the condition that the number

$$h(a,b,c,d) = m(a,b,d)m(a,c,d)m(b,a,c)m(b,d,c) \quad 2.4.2$$

is zero. A design defined by a density *p* that is element wise constant on a mesh of quadrilaterals with *N* interior nodes will thus be free of checkerboards if it satisfies the constraint

$$\sum_{k=1}^{N} h(\rho_{k,a}, \rho_{k,b}, \rho_{k,c}, \rho_{k,d}) = 0 \quad 2.4.3$$

Where $\rho_{k,e}, e = a, b, c, d$ is the material densities in the elements connected to the node *k*. This constraint can thus be added to our optimization problem to assure checkerboard free solutions. It can also be used to remove "artificial" hinges in mechanism design. As we have seen in other situations, an implementation using gradient based optimization techniques requires a replacement of the absolute value by a smooth substitute, for example $|x| \simeq \sqrt{x^2 + \epsilon^2} - \epsilon$ with ϵ = 0.1. With this modification a sensitivity analysis of the constraint is straightforward, but rather tedious

2.5 Filtering of sensitivities technique

Filters are used to prevent checkerboarding by smoothening the stiffness in a fashion similar to the filtering of an image. Filtering meant that stiffness in a point e depends on the density x_e in all points in the neighborhood of *e*. The method gives existence of solutions and convergence with refinement of FE mesh. Filtering the sensitivity information of the optimization problem is an efficient way to ensure mesh-independency. Filtering works by modifying the density sensitivity of a specific element based on weighted average of the element sensitivities in a fixed neighborhood. The scheme works by modifying the element sensitivity of the compliance as:

$$\frac{\partial \hat{c}}{\partial x_e} = \frac{1}{x_e \sum_{i=1}^{N} \hat{H}_i} \sum_{i=1}^{N} \hat{H}_i x_i \frac{\partial c}{\partial x_i} , \quad 2.5.1$$

Where N is the total number of element in the mesh and where the mesh-independent convolution operator (weight factor) $\hat{H}_i$ is

$$\hat{H}_i = r_{min} - dist(e,i), \{i \in N \mid dist\,(e,i) \le r_{min}\}, \;\; e = 1, \ldots, N \;. \quad 2.5.2$$

The operator *dist (e,i)* is the distance between the center of element *e* and the center of element *i* . The convolution operator $\hat{H}_i$ is zero outside the filter area. In the case of a linear filter the convolution operator for element *i* decay linearly with distance from element e. Other filters that can be used are the so-called non-linear and 3-by-3 filters.

III. COMPARISON OF METHODS

To overcome checkerboard problem, use of high order finite element leads to a more expensive computer problem and, sometimes, cannot even solve the problem if SIMP exponent higher than 3.

Perimeter constraint is a good solution, because we are not only solving the checkerboard pattern but also the mesh dependency problem. Thus, constraining the perimeter, we can avoid the formation of several small holes (voids between two solid elements in a checkerboard pattern, for example). Two drawbacks can be noted in this formulation. The first and more direct is that we are adding a new constraint in the optimization problem, and manage with many constraints usually is not an easy task. The second one is that, a priori, we have no idea about which amount of perimeter we have to constraint. This can lead to different final topologies.
The perimeter and filter methods produce very similar designs, but there are some differences. The perimeter control is global constraints and will allow the formation of locally very thin bars. The filtering schemes will generally remove thin bars. Predicting the value of the perimeter constraint for a new design problem must be determined by experiments, since there is no direct relation between local scale in the structure and the perimeter bound. If the perimeter bound is too tight, there may be no solution to the optimization problem. This problem is particularly difficult for three-dimensional problem.

IV. FUTURE WORK

Insight into the checkerboarding helps us to improve the mathematical instability. The future work can be in the field of algorithm development or the refinement of available methods so that the effect of instability can be reduced. Mathematical instability seems to be small when we deal with the simple models but in case of complex models both time and energy can be saved if more advanced algorithm can be developed.

V. CONCLUSIONS

Of the four methods, the higher-order finite elements method is probably the most convenient one. If checkrboarding is to be controlled irrespective of problem higher-order finite element is used. Computational cost of high-order finite element method is high for complex structures so most of the software based algorithms are taking this method as a optional method. No external techniques are needed other than altering the element that is used to discretize the design domain to higher-order finite element. When the checkerboarding problem comes up as per the need methods should be changed to get final Topology Optimization. Finally, we remark that theoretical studies of the appearance of checkerboards in three-dimensional problems are yet to be carried out. However numerical experience shows that checkerboards also appear for this case.

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