SEPARATION AXIOMS USING IG*, DG*, BG*- CLOSED TYPE SETS IN TOPOLOGICAL ORDERED SPACES

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ABSTRACT

In this paper we discuss possible applications of ig,dg* & bg*-closed type sets in Topological ordered spaces.*

KEY WORDS: *Topological ordered space, closed set, g-closed set and g*-closed set*.

I. INTRODUCTION

Leopoldo Nachbin [1] initiated the study of topological ordered spaces. Levine [4] introduced the class of g-closed sets, a super class of sets in 1970. M.K.R.S. Veera Kumar [2]introduced a new class of sets, called g*-closed sets in 2000, which is properly placed in between the class of closed sets and the class of g-closed sets M.K.R.S. Veera Kumar [3] introduced the study of i-closed, d-closed and b-closed sets in 2001. G.Srinivasarao introduced the study of ig-closed, dg-closed, bg-closed, ig*-closed, dg*-closed and bg*-closed sets in 2014[5].

II. PRELIMINARIES

Definition 2.1 A subset A of a topological space (X , τ , $\leq$) is called

1. An **i-closed** set [3] if A is an increasing set and closed set.
2. A **d-closed** set [3] if A is a decreasing set and closed set.
3. A **b-closed** set [3] if A is a both increasing and decreasing set and a closed set.
4. **ig-closed** set [5] if icl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
5. **dg-closed** [5] set if dcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
6. **bg-closed** set [5] if bcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

Theorem 2.2. [2] Every closed set is a g-closed set.

The following example supports that a g-closed set need not be closed set in general.

Example 2.3. [2] Let X = {a , b , c} , □▫□= {□□, X , {a}} and ≤1 = {(a , a) , (b , b) ,(c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , , □▫□□≤1) is a topological ordered space. closed sets are □□, X , {b , c}. g-closed sets are □□, X , {b}, {c}, {a,b}, {b , c}, {c,a}.Let A={c}. Clearly A is a g-closed set but not a closed set.

Theorem 2.4. [2] Every g*-closed set is a g-closed set.

The following example supports that a g-closed set need not be a g*-closed set in general.

Example 2.5. [2] Let X = {a , b , c} , 2 □□= {□□, X , {a}} and ≤1 = {(a , a) , (b , b) ,(c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , □▫▫▫▫□□≤1) is a topological ordered spaces. g-closed sets are□□, X , {b}, {c}, {a,b}, {b , c}, {c,a}. g*-closed sets are □□, X , {b , c}.Let A={c}. Then A is a g-closed set but not a g*-closed set.

Theorem 2.6. [5] Every i-closed set is an ig-closed set.

The following example supports that an ig-closed set need not be an i-closed set in general.

Example 2.7. [5] Let X = {a , b , c} , 2 □□= {□□, X , {a}} and ≤2 = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , □□□□□□≤2) is a topological ordered space. ig-closed sets are Φ , X , {b} , {a , b}. i-closed sets are □□, x. Let A = {b} or {a , b}. Clearly A is an ig-closed set but not an i-closed set.

Theorem 2.8.[5] Every d-closed set is a dg-closed set.

The following example supports that a dg-closed set need not be d-closed set in general.

Example 2.9. [5] Let X = {a , b , c} , 2 □□= {□□, X , {a}} and ≤2 = {(a , a) , (b , b) , (c , c) ,(a , b) , (c , b)}. Clearly (X , □□□□□□≤2) is a topological ordered space.dg-closed sets are□□, X , {c} , {b , c}. d-closed sets are □□, X , {b , c}.Let A = {c}. Clearly A is a dg-closed set but not a d-closed set.

Theorem 2.10. [5] Every b-closed set is a bg-closed set.

The following example supports that a bg-closed set need not be a b-closed set in general.

Example 2.11. [5] Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , τ_2, $\leq_3$) is a topological ordered space. bg-closed sets are ϕ , X , {c}. b-closed sets are ϕ , X. Let A = {c}.Clearly A is a bg-closed set but not a b-closed set.

Theorem 2.12. [5] Every bg-closed set is an ig-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

Example 2.13. [5] Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , τ_1, $\leq_1$) is a topological ordered space

Let A = {c}. Clearly A is an ig closed set but not a bg-closed set.

THEOREM 2.14. [5] Every bg-closed set is a dg-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.15. [5] Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , τ_1, $\leq_3$) is a topological ordered space.

Let A ={a , c}. Clearly A is a dg-closed set but not a bg-closed set.

THEOREM 2.16. [5] Every b-closed set set is an i-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.17. [5] Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , τ_1, $\leq_1$) is a topological ordered space. i-closed sets are ϕ , X , {c} , {b , c}.b-closed sets are ϕ , X. Let A = {c} or {b , c} . Clearly A is an i-closed set but not a b-closed set.

THEOREM 2.18.[5] Every b-closed set is a d-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.19. [5] Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} an $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_1, $\leq_2$) is a topological ordered space.d-closed sets are ϕ , X , {c} , {b , c}. b-closed sets are ϕ , X. Let A = {c} or {b , c}. Clearly A is a d-closed set but not a b-closed set.

THEOREM 2.20. [5] Every ig*-closed set is an ig-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.21. [5] Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , τ_2, $\leq_1$) is a topological ordered space. ig-closed sets are ϕ , X , {c} , {b , c}. ig*-closed sets are ϕ, X , {b , c}. Let A = {c}. Clerly A is a ig-closed set but not a ig*-closed set.

THEOREM 2.22. [5] Every dg*-closed set is an dg-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.23. **[5]** Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_2$ = {(a , a) , (b , b), (c , c) , (a , b) , (c , b)}. Clearly (X , τ_2, $\leq_2$) is a topological ordered space.dg-closed sets are ϕ , X , {c} , {b , c}. dg*-closed sets are ϕ , X , {b , c}.Let A = {c}. Clearly A is an dg-closed set but not a dg*-closed set.So the class of dg-closed sets properly contains the class of all dg*-closed sets.

THEOREM 2.24. [5] Every bg*-closed set is a bg-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.25. [5] Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}.Clearly (X , τ_2, $\leq_3$) is a topological ordered space. bg*-closed sets are ϕ , X. bg-closed sets are ϕ , X , {c}. Let A = {c}. Clearly A is bg-closed set but not a bg*-closed set. So the class of bg-closed sets properly contains the class of all bg*-closed sets.

THEOREM 2.26. [5] Every bg*-closed set is an ig*-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.27. [5] : Let X = {a , b , c} , τ_3 = { ϕ , X , {a} , {b , c}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , τ_3, $\leq_3$) is a topological ordered space.
Let A = {b}. Clearly A is an ig*-closed set but not a bg*-closed set.

THEOREM 2.28. [5] Every bg*-closed set is an dg*-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.29. **[5]** Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , τ_1, $\leq_3$) is a topological ordered space. Let A = {a , c}. Clearly A is a dg*-closed set but not a ig*-closed set. The class of all dg*-closed sets properly contains the class of all bg*-closed sets.

THEOREM 2.30.[5] Every i-closed set is an ig*-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.31. [5] Let X = {a , b , c} , τ_3 = { ϕ , X , {a} , {b , c}} and $\leq_4$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_3, $\leq_4$) is a topological ordered space. ig*-closed sets are ϕ , X , {b , c}. i-closed sets are ϕ , X.Let A = {b , c}. Clearly A is a ig*-closed set but not an i-closed set.The class of all ig*-closed sets properly contains the class of all i-closed sets.

THEOREM 2.32. [5] Every d-closed set is a dg*-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.32. [5] Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_4, $\leq_2$) is a topological ordered space. dg*-closed sets are ϕ , X , {b , c}. d-closed sets are ϕ , X.Let A = {b , c}. Then A is dg*-closed set but not a d-closed set. The class of all dg*-closed sets properly contains the class of all d-closed sets.

THEOREM 2.33. [5] Every b-closed set is a bg*-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.34. [5] Let X = {a , b , c} , τ_6 = { ϕ , X , {a} , {b} , {a , b}{a , c}} and $\leq_7$ = {(a , a), (b , b), (c , c), (b , c), (c , a), (b , a)}. Clearly (X , τ_6, $\leq_7$) is a topological ordered space.bg*-closed sets are ϕ , X , {b}. b-closed sets are ϕ , X. Let A = {b}. Then A is bg*-closed set but not a b-closed set. The class of all bg*-closed sets properly contains the class of all b-closed sets.

THEOREM 2.35. [5] Every bg*-closed set is an ig-closed set.

Then every bg*-closed set is an ig-closed set. The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.36. [5] Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , τ_1, $\leq_3$) is a topological ordered space.bg*-closed sets are ϕ ,

X. ig-closed sets are ϕ , X , {c} , {b , c}. Let A = {c} or {b , c}. Clearly A is an ig-closed set but not a bg^*-closed set.The class of all ig-closed sets properly contains the class of all bg^*-closed sets.

THEOREM 2.37. [5] Every bg^*-closed set is a dg-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 2.38. [5] Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}.Clearly (X , $\tau_1, \leq_2$) is a topological ordered space.bg^*-closed sets are ϕ , X. dg-closed sets are ϕ , X , {c} , {b , c}. Let A = {c} or {b , c}. Clearly A is a dg-closed set but not a bg^*-closed set.

III. APPLICATIONS OF G*-CLOSED TYPE SETS

DEFINITION 3.1. [5] A subset 'A' of (X , τ, $\leq$) is called a **ig^*-closed set** if icl(A) $\subseteq$ U whenever A $\subseteq$ U and U is a g-open in (X , τ).The class of all ig^*-closed subsets of (X , τ) is denoted by $IG^*C(X)$.

DEFINITION 3.2.[5] A subset 'A' of (X , τ, $\leq$) is called a **dg^*-closed** set if dcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is a g-open in (X , τ).The class of all dg^*-closed subsets of (X , τ) is denoted by $DG^*C(X)$.

DEFINITION 3.3. [5] A subset 'A' of (X , τ, $\leq$) is called a **bg^*-closed** set if bcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is a g-open in (X , τ).The class of all dg^*-closed subsets of (X , τ) is denoted by $BG^*C(X)$.

DEFINITION 3.4 A topological ordered space (X, τ , $\leq$) is called

i) a **${}_iT_{1/2}^*$space**, if every ig*-closed set is closed.
ii) a **${}_dT_{1/2}^*$space**, if every dg*-closed set is closed.
iii) a **${}_bT_{1/2}^*$space**, if every bg*-closed set is closed.

DEFINITION 3.5. A topological ordered space (X, τ , $\leq$) is called

i) **${}_iT_i{}^*{}_{1/2}$** space if every ig*-closed set is an i-closed set.
ii) **${}_dT_d{}^*{}_{1/2}$** space if every dg*-closed set is a d-closed set.
iii) **${}_bT_b{}^*{}_{1/2}$** space if every bg*-closed set is a b-closed set.

THEOREM 3.6. Every ${}_iT_{1/2}^*$ space is ${}_bT_{1/2}^*$ space.

Proof: Let (X , τ) be ${}_iT_{1/2}^*$ space. We show that (X , τ) is ${}_bT_{1/2}^*$ space.Let A be a bg*-closed subset of X. Then A is an ig*-closed subset of A.Since (X , τ) is an ${}_iT_{1/2}^*$ - space then A is closed. Every bg*-closed subset of X is a closed set. Hence (X , τ) is ${}_bT_{1/2}^*$ space. Thus every ${}_iT_{1/2}^*$ space is ${}_bT_{1/2}^*$ space. The converse of the above theorem need not be true. This will be justify from the following example.

EXAMPLE 3.7. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , $\tau_4, \leq_2$) is a topological ordered space. bg*-closed sets are ϕ , X . closed sets are ϕ , X , {a} , {b , c}. ig*-closed sets are ϕ , X , {b} , {a , b}. Here every bg*-closed set is a closed set. Therefore (X , τ_2, $\leq_3$) is ${}_bT_{1/2}^*$ space. Let A = {a , b}. Clearly A is an ig*-closed set but not a closed set. Hence (X , τ_2, $\leq_3$) is not a ${}_iT_{1/2}^*$ space.

THEOREM 3.8. Every ${}_dT_{1/2}^*$ space is ${}_bT_{1/2}^*$ space.

Proof: Let (X , τ) be ${}_dT_{1/2}^*$ space. We show that (X , τ) is ${}_bT_{1/2}^*$ space.Let A be a bg*-closed subset of X. Then A is an dg*-closed subset of A.Since (X , τ) is an ${}_dT_{1/2}^*$ - space then A is closed. Every bg*-closed subset of X is a closed set.Hence (X , τ) is ${}_bT_{1/2}^*$ space. Thus every ${}_dT_{1/2}^*$ space is ${}_bT_{1/2}^*$ space.The converse of the above theorem need not be true. This will be justify from the following example.

EXAMPLE 3.9. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , $\tau_4, \leq_3$) is a topological ordered space. bg*-closed sets are ϕ , X . closed sets are ϕ , X , {a} , {b , c}. dg*-closed sets are ϕ , X , {b} . Here every bg*-closed set is a closed set. Therefore (X , τ_4, $\leq_3$) is $_bT_{1/2}^*$ space. Let A = { b}. Clearly A is an dg*-closed set but not a closed set. Hence (X , τ_4,$\leq_3$) is not a $_dT_{1/2}^*$ space.

THEOREM 3.10. $_iT_{1/2}^*$ space and $_dT_{1/2}^*$ space are independent notions. This will be seen in the following examples.

EXAMPLE 3.11. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_4,$\leq_2$) is a topological ordered space. bg*-closed sets are ϕ , X . closed sets are ϕ , X , {a} , {b , c}. ig*-closed sets are ϕ , X , {b} , {a , b}. Here every bg*-closed set is a closed set. Therefore (X , τ_2, $\leq_3$) is $_bT_{1/2}^*$ space. Let A = {a , b}. Clearly A is an ig*-closed set but not a closed set. Hence (X , τ_2, $\leq_3$) is not a $_iT_{1/2}^*$ space.

EXAMPLE 3.12. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , τ_4,$\leq_3$) is a topological ordered space. bg*-closed sets are ϕ , X . closed sets are ϕ , X , {a} , {b , c}. dg*-closed sets are ϕ , X , {b} . Here every bg*-closed set is a closed set. Therefore (X , τ_4, $\leq_3$) is $_bT_{1/2}^*$ space. Let A = { b}. Clearly A is an dg*-closed set but not a closed set. Hence (X , τ_4,$\leq_3$) is not a $_dT_{1/2}^*$ space.

THEOREM 3.13. Every $_iT_{i,1/2}$ space is an $_iT_{i,}*_{1/2}$ space.

Proof : Let (X , τ , $\leq$) be an $_iT_{i,1/2}$ space . We show that (X , τ , $\leq$) is an $_iT_{i,}*_{1/2}$ space. Let A be an ig*-closed subset of X. Then A is an ig-closed subset of X. Since (X , τ , $\leq$) be an $_iT_{i,1/2}$ space and A is an ig-closed subset of X, then A is an i-closed set. Every ig*-closed set is an i-closed set. Then (X , τ , $\leq$) is an $_iT_{i,}*_{1/2}$ space. Thus every $_iT_{i,1/2}$ space is an $_iT_{i,}*_{1/2}$ space. The converse of the above theorem need not be true. This will be justfy from the following example.

EXAMPLE 3.14. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_2$ = {(a , a) , (b , b) ,(c , c) , (a , b) , (c , b)}. Clearly (X , τ_2,$\leq_2$) is a topological ordered space. ig-closed sets are ϕ , X , {b} , {a , b}. ig*-closed sets are ϕ , X . i-closed sets are ϕ , X . Here every ig*-closed set is an i-closed set . Let A = {b} or {a , b}. Clearly A is an ig-closed set but not an i- closed set. Hence (X , τ_2,$\leq_2$) is an $_iT_{i,}*_{1/2}$ space and not a $_iT_{i,1/2}$ space.

THEOREM 3.15. Every $_dT_{d\,,\,1/2}$ space is an $_dT_{d\,,}*_{1/2}$ space.

Proof: Let (X , τ , $\leq$) be a $_dT_{d\,,\,1/2}$ space . We show that (X , τ , $\leq$) is an $_dT_{d\,,}*_{1/2}$ space. Let A be a dg*-closed subset of X. Then A is a dg-closed subset of X.Since (X , τ , $\leq$) be an $_dT_{d\,,\,1/2}$ space and A is a dg-closed subset of X, then A is a d-closed set.Every dg*-closed set is a d-closed set. Then (X , τ , $\leq$) is an $_dT_{d\,,}*_{1/2}$ space. Thus every $_dT_{d\,,\,1/2}$ space is a $_dT_{d\,,}*_{1/2}$ space. The converse of the above theorem need not be true. This will be justify from n the following example.

EXAMPLE 3.16. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_6$ = {(a , a) , (b , b) ,(c , c) , (b , a) , (a , c) , (b , c)}. Clearly (X , τ_2,$\leq_6$) is a topological ordered space. dg-closed sets are ϕ , X , {b} , {a , b}. dg*-closed sets are ϕ , X . d-closed sets are ϕ , X . Here every dg*-closed set is a d-closed set . A = {b} or {a , b}. Clearly A is a dg-closed set but not a d- closed set. Hence (X , τ_2,$\leq_6$) is an $_dT_{d\,,}*_{1/2}$ space and not a $_dT_{d\,,\,1/2}$ space.

THEOREM 3.17. Every $_bT_{b\,,\,1/2}$ space is an $_bT_{b\,,}*_{1/2}$ space.

Proof: Let (X , τ , $\leq$) be a $_bT_{b\,,\,1/2}$ space . We show that (X , τ , $\leq$) is an $_bT_{b\,,}*_{1/2}$ space. Let A be a bg*-closed subset of X. Then A is a bg-closed subset of X. Since (X , τ , $\leq$) be an $_bT_{b\,,\,1/2}$ space and A is a bg-closed subset of X, then A is a b-closed set. Every bg*-closed set is a b-closed set. Then (X , τ , $\leq$) is an $_bT_{b\,,}*_{1/2}$ space. Thus every $_bT_{b\,,\,1/2}$ space is a $_bT_{b\,,}*_{1/2}$ space. The converse of the above theorem need not be true. This will be justify from the following example.

EXAMPLE 3.18. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_3$ = {(a , a) , (b , b) ,(c , c) , (a , b) , (a , c)}. Clearly (X , τ_2,$\leq_3$) is a topological ordered space. bg-closed sets are ϕ , X , {b , c} . bg*-closed sets are ϕ , X . b-closed sets are ϕ , X . Here every bg*-closed set is a b-closed set .

Let A = {b} or {a , b}. Clearly A is a bg-closed set but not a b- closed set. Hence (X , $\tau_2, \leq_3$) is an $_bT_{b}$,$*_{1/2}$ space and not a $_bT_{b,1/2}$ space.

THEOREM 3.19. The spaces $_dT_{d}$, $*_{1/2}$ and $_bT_{b}$, $*_{1/2}$ are independent notions. This will be seen in the following examples.

EXAMPLE 3.20. Let X = {a , b , c} , τ_6, = { ϕ , X , {a} , {b} , {a,b} , {a , c}} and $\leq_7$ = {(a , a) , (b , b) , (c , c) ,(b , c) , (c , a) , (b , a)}. Clearly (X , τ_6, $\leq_7$) is a topological ordered space. Here d-closed sets are ϕ , X , {a , c} . dg*- closed sets are ϕ , X , {a , c} and b-closed sets are ϕ , X bg*-closed sets are ϕ , X , {b} . Clearly every ig-closed set is an i-closed set. So(X , τ_6, $\leq_7$) is $_dT_{d}$,$*_{1/2}$ space. Let A = {b} . Clearly A is a bg*-closed set but not a b-closed set. Thus (X , $\tau_6, \leq_{67}$) is not a $_bT_{b}$,$*_{1/2}$ space.

EXAMPLE 3.21. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , $\tau_4, \leq_2$) is a topological ordered space. bg*-closed sets are ϕ , X . b-closed sets are ϕ , X . dg*-closed sets are ϕ , X , {b , c} . Here every bg*-closed set is a closed set. Therefore (X , τ_2, $\leq_3$) is $_bT_{1/2}^*$ space. Let A = {b , c}. Clearly A is a bg*-closed set but not a b-closed set. Hence (X , τ_2, $\leq_3$) is not a $_bT_{b}$,$^*{}_{1/2}$ space.

THEOREM 3.22. Every $_iT_b$ space is an $_bT_b$,$*_{1/2}$ space.

Proof: Let (X , τ , $\leq$) be a $_iT_b$ space . We show that (X , τ , $\leq$) is an $_bT_b$,$*_{1/2}$ space. Let A be a bg*-closed subset of X. We know that every balanced set is an increasing set and g*-closed set is a g-closed set. Then A is an ig-closed subset of X. Since (X , τ , $\leq$) be an $_iT_b$ space and A is an ig-closed subset of X, then A is a b-closed set. Every bg*-closed set is a b-closed set. Then (X , τ , $\leq$) is an $_bT_b$, $*_{1/2}$ space. Thus every $_bT_{b,1/2}$ space is a $_bT_b$, $*_{1/2}$ space.The converse of the above theorem need not be true. This will be justify from the following example.

EXAMPLE 3.23. Let X = {a , b , c} , τ_1, = { ϕ , X , {a},{b},{a,b}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b,c),(a,c)}. Here i-closed sets are ϕ , X , {c},{b,c}. b-closed sets are ϕ , X and bg*-closed sets are ϕ , X . Let A = {c} or {b , c}. Clearly A is an ig-closed set but not a b-closed set. Every bg*-closed set is a b-closed set. Thus (X , $\tau_1, \leq_1$) is a $_bT_{b}$, $*_{1/2}$ space but not $_iT_b$ space.

THEOREM 3.24. Every $_dT_b$ space is an $_bT_b$,$*_{1/2}$ space.

Proof: Let (X , τ , $\leq$) be a $_dT_b$ space . We show that (X , τ , $\leq$) is an $_bT_b$,$*_{1/2}$ space. Let A be a bg*-closed subset of X. We know that every balanced set is a decreasing set and every g*-closed set is a g-closed set. Then A is a dg-closed subset of X. Since (X , τ , $\leq$) be an $_dT_b$ space and A is a dg-closed subset of X, then A is a b-closed set.Every bg*-closed set is a b-closed set. Then (X , τ , $\leq$) is an $_bT_b$, $*_{1/2}$ space. Thus every $_dT_b$ space is a $_bT_b$, $*_{1/2}$ space. The converse of the above theorem need not be true. This will be justify from the the following example.

EXAMPLE 3.25. Let X = {a , b , c} , τ_1, = { ϕ , X , {a},{b},{a,b}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b,c),(a,c)}. Here dg-closed sets are ϕ , X , {c},{b,c}. b-closed sets are ϕ , X and bg*-closed sets are ϕ , X . Let A = {c} or {b , c}. Clearly A is a dg-closed set but not a b-closed set. Every bg*-closed set is a b-closed set. Thus (X , $\tau_1, \leq_1$) is a $_bT_{b}$, $*_{1/2}$ space but not $_dT_b$ space.

THEOREM 3.26. Every $_iT_{1/2}$ space is $_bT_{1/2}^*$ space.

Proof: Let (X , τ , $\leq$) be an $_iT_{1/2}$ space . We show that (X , τ , $\leq$) is an $_bT*_{1/2}$ space.
Let A be a bg*-closed subset of X. Then A is an ig-closed subset of X. Since (X , τ , $\leq$) be an $_iT_{1/2}$ space and A is an ig-closed subset of X, then A is a closed set. Every bg*-closed set is a closed set. Then (X , τ , $\leq$) is an $_bT_{1/2}^*$ space. Thus every $_iT_{1/2}$ space is an $_bT_{1/2}{}^*$ space. The converse of the above theorem need not be true. This will be justify from the the following example.

EXAMPLE 3.27. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_2$ = {(a , a) , (b , b) ,(c , c) , (a , b) , (c , b)}. Clearly (X , $\tau_2, \leq_2$) is a topological ordered space. ig-closed sets are ϕ , X , {b} , {a , b} . bg*-closed sets are ϕ, X , {b , c}. closed sets are ϕ, X , {b , c}. Here every bg*-closed set is a closed set . Let A = {b} or {a , b}. Clearly A is an ig-closed set but not an closed set. Hence (X , $\tau_2, \leq_3$) is an $_bT_{1/2}$* space and not a $_iT_{1/2}$ space.

THEOREM 3.28. Every $_dT_{1/2}$ space is $_bT_{1/2}^*$ space.

Proof: Let (X , τ , ≤) be an $_dT_{1/2}$ space . We show that (X , τ , ≤) is an $_bT^*_{1/2}$ space.
Let A be a bg*-closed subset of X. Then A is a dg-closed subset of X. Since (X , τ , ≤) be an $_dT_{1/2}$ space and A is a dg-closed subset of X, then A is a closed set. Every bg*-closed set is a closed set. Then (X , τ , ≤) is an $_bT_{1/2}^*$ space. Thus every $_dT_{1/2}$ space is an $_bT_{1/2}^*$ space. The converse of the above theorem need not be true. This will be justify from the the following example.

EXAMPLE 3.29. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_2$ = {(a , a) , (b , b) ,(c , c) , (a , b) , (c , b)}. Clearly (X , $\tau_2, \leq_2$) is a topological ordered space. ig-closed sets are ϕ , X , {b} , {a , b} . bg*-closed sets are ϕ , X , {b , c}. closed sets are ϕ , X , {b , c}. Here every bg*-closed set is a closed set . Let A = {b} or {a , b}. Clearly A is an ig-closed set but not an closed set. Hence (X , $\tau_2, \leq_3$) is an $_bT_{1/2}$* space and not a $_iT_{1/2}$ space.

THEOREM 3.30. The spaces $_dT_{1/2}$ * and $_iT_{1/2}$ are independent notions. This will be seen in the following examples.

EXAMPLE 3.31. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_1$ = {(a , a) , (b , b) ,(c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , $\tau_2, \leq_1$) is a topological ordered space. ig-closed sets are ϕ , X , {c} , {b , c} . dg*-closed sets are ϕ , X . closed sets are ϕ , X , {b , c}. Here every dg*-closed set is a closed set . Let A = {c} . Clearly A is an ig-closed set but not an closed set. Hence (X , $\tau_2, \leq_1$) is an $_iT_{1/2}$ space and not a $_dT_{½}^*$ space.

EXAMPLE 3.32. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_4, $\leq_3$) is a topological ordered space. dg*-closed sets are ϕ , X , {a , b}. closed sets are ϕ , X , {b} , {b , c) . ig-closed sets are ϕ , X , {b} . Here every ig-closed set is a closed set. Therefore (X , τ_2, $\leq_3$) is $_iT_{1/2}$ space. Let A = {a , b} Clearly A is a dg*-closed set but not a closed set. Hence (X , τ_2, $\leq_3$) is not a $_dT_{½}^*$ space.

THEOREM 3.33. The spaces $_dT_{1/2}$ * and $_bT_{1/2}$ are independent notions. This will be seen in the following examples.

EXAMPLE 3.34. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_3$ = {(a , a) , (b , b) ,(c , c) , (a , b) , (a , c)}. Clearly (X , τ_2, $\leq_3$) is a topological ordered space. bg-closed sets are ϕ , X , {c}. dg*-closed sets are ϕ , X . closed sets are ϕ , X , {b} , {b , c}. Here every dg*-closed set is a closed set . Let A = {c} . Clearly A is a bg-closed set but not an closed set. Hence (X , τ_2, $\leq_1$) is an $_dT_{1/2}^*$ space and not a $_bT_{1/2}$ space.

EXAMPLE 3.35. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , $\tau_4, \leq_3$) is a topological ordered space. dg*-closed sets are ϕ , X , {a , b}. closed sets are ϕ , X , {b} , {b , c) . bg-closed sets are ϕ , X . Here every bg-closed set is a closed set. Therefore (X , τ_2, $\leq_3$) is $_bT_{1/2}$ space. Let A = {a , b} Clearly A is a dg*-closed set but not a closed set. Hence (X , τ_2, $\leq_3$) is not a $_dT_{1/2}^*$ space.

THEOREM 3.36. The spaces $_dT_{d,1/2}$ and $_bT_{b^*,1/2}$ are independent notions. This will be seen in the following examples.

EXAMPLE 3.37. Let X = {a , b , c} , τ_6, = { ϕ , X , {a} , {b} , {a,b} , {a , c}} and $\leq_7$ = {(a , a) , (b , b) , (c , c) ,(b , c) , (c , a) , (b , a)}. Clearly (X , τ_6, $\leq_7$) is a topological ordered space. Here d-closed sets are ϕ , X , {b} , {b , c} . dg-closed sets are ϕ , X , {b} , {b , c} and b-closed sets are ϕ , X bg*- closed sets are ϕ , X , {b} . Clearly every dg-closed set is a d-closed set. So(X ,

τ_6, $\leq_7$) is $_dT_{d,1/2}$ space. Let A = {b} . Clearly A is a bg*-closed set but not a b-closed set. Thus (X , τ_6, $\leq_7$) is not a $_bT_{b,}*_{1/2}$ space.

EXAMPLE 3.38. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_4, $\leq_2$) is a topological ordered space. dg-closed sets are ϕ , X , {b , c}. d-closed sets are ϕ, X. bg*-closed sets are ϕ, X . b-closed sets are ϕ, X Here every bg*-closed set is a b-closed set. Therefore (X , τ_2, $\leq_3$) is $_bT_b{}^*{}_{1/2}$ space. Let A = {a , b} Clearly A is a dg-closed set but not a d-closed set. Hence (X , τ_2, $\leq_3$) is not a $_dT_{d,1/2}$ space.

THEOREM 3.39. The spaces $_dT_{d,1/2}$ and $_bT_{b^*,1/2}$ are independent notions. This will be seen in the following examples.

EXAMPLE 3.40. Let X = {a , b , c} , τ_6, = { ϕ , X , {a} , {b} , {a,b} , {a , c}} and $\leq_7$ = {(a , a) , (b , b) , (c , c) ,(b , c) , (c , a) , (b , a)}. Clearly (X , τ_6, $\leq_7$) is a topological ordered space. Here d-closed sets are ϕ , X , {b} , {b , c} . dg-closed sets are ϕ , X , {b} , {b , c} and b-closed sets are ϕ , X bg*- closed sets are ϕ , X , {b} . Clearly every dg-closed set is a d-closed set. So(X , τ_6, $\leq_7$) is $_dT_{d,1/2}$ space. Let A = {b} . Clearly A is a bg*-closed set but not a b-closed set. Thus (X , τ_6, $\leq_7$) is not a $_bT_{b,}*_{1/2}$ space.

EXAMPLE 3.41. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_4, $\leq_2$) is a topological ordered space. dg-closed sets are ϕ , X , {b , c}. d-closed sets are ϕ, X. bg*-closed sets are ϕ, X . b-closed sets are ϕ, X Here every bg*-closed set is a b-closed set. Therefore (X , τ_2, $\leq_3$) is $_bT_b{}^*{}_{1/2}$ space. Let A = {a , b} Clearly A is a dg-closed set but not a d-closed set. Hence (X , τ_2, $\leq_3$) is not a $_dT_{d,1/2}$ space.

THEOREM 3.42. The spaces $_iT_i{}^*{}_{1/2}$ and $_dT_{d^*,1/2}$ are independent notions. This will be seen in the following examples.

EXAMPLE 3.43. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_4, $\leq_3$) is a topological ordered space. dg*-closed sets are ϕ , X , {a , b}. d-closed sets are ϕ , X , {b} , {b , c) . bg*-closed sets are ϕ , X , {b}. b-closed sets are ϕ , X . Here every bg*-closed set is a b-closed set. Therefore (X , τ_2, $\leq_3$) is $_bT_{b,}{}^*{}_{1/2}$ space. Let A = {b , c} Clearly A is a dg*-closed set but not a d-closed set. Hence (X , τ_2, $\leq_3$) is not a $_dT_d{}^*{}_{1/2}$ space.

EXAMPLE 3.44. Let X = {a , b , c} , τ_6, = { ϕ , X , {a} , {b} , {a,b} , {a , c}} and $\leq_7$ = {(a , a) , (b , b) , (c , c) ,(b , c) , (c , a) , (b , a)}. Clearly (X , τ_6, $\leq_7$) is a topological ordered space. Here d-closed sets are ϕ , X , {b} , {b , c} . dg*-closed sets are ϕ , X , {b , c} and b-closed sets are ϕ , X bg*- closed sets are ϕ , X , {b} . Clearly every dg-closed set is a d-closed set. So(X , τ_6, $\leq_7$) is $_dT_{d,}{}^*{}_{1/2}$ space. Let A = {b} . Clearly A is a bg*-closed set but not a b-closed set. Thus (X , τ_6, $\leq_7$) is not a $_bT_{b,}*_{1/2}$ space.

THEOREM 3.45. The spaces $_iT_i{}^*{}_{1/2}$ and $_bT_{b^*,1/2}$ are independent notions. This will be seen in the following examples.

EXAMPLE 3.46. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_4$ = {(a , a) , (b , b) , (c , c) , (c , a) , (c , b)}. Clearly (X , τ_4, $\leq_4$) is a topological ordered space. ig*-closed sets are ϕ , X , {b} , {a , b}. i-closed sets are ϕ , X , {a , b}. bg*-closed sets are ϕ , X. b-closed sets are ϕ , X . Here every bg*-closed set is a b-closed set. Therefore (X , τ_2, $\leq_3$) is $_bT_{b,}{}^*{}_{1/2}$ space. Let A = {b } Clearly A is an ig*-closed set but not an i-closed set. Hence (X , τ_2, $\leq_3$) is not a $_iT_i{}^*{}_{1/2}$ space.

EXAMPLE 3.47. Let X = {a , b , c} , τ_6, = { ϕ , X , {a} , {b} , {a,b} , {a , c}} and $\leq_7$ = {(a , a) , (b , b) , (c , c) ,(b , c) , (c , a) , (b , a)}. Clearly (X , τ_6, $\leq_7$) is a topological ordered sp b-closed sets are ϕ , X bg*- closed sets are ϕ , X , {b} . ig*-closed sets are ϕ , X , {c , a}. i-closed sets are ϕ , X , {a , c}. bg*-closed sets are ϕ , X. b-closed sets are ϕ , X. Clearly every ig*-closed set is an i-closed set. So(X , τ_6, $\leq_7$) is ${}_iT_{i,}{}^*{}_{1/2}$ space. Let

A = {b} . Clearly A is a bg*-closed set but not a b-closed set. Thus (X , τ_6, $\leq_7$) is not a ${}_bT_{b,}{}^*{}_{1/2}$ space.

THEOREM 3.48. The spaces ${}_iT_i{}^*{}_{1/2}$ and ${}_dT_{d,,1/2}$ are independent notions. This will be seen in the following examples.

EXAMPLE 3.49. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_4$ = {(a , a) , (b , b) , (c , c) , (c , a) , (c , b)}. Clearly (X , τ_4, $\leq_4$) is a topological ordered space. ig*-closed sets are ϕ , X , {b} , {a , b}. i-closed sets are ϕ , X , {a , b}. dg-closed sets are ϕ , X. d-closed sets are ϕ , X . Here every dg-closed set is a d-closed set. Therefore (X , τ_2, $\leq_3$) is ${}_dT_{d,1/2}$ space. Let A = {b } Clearly A is an ig*-closed set but not an i-closed set. Hence (X , τ_2, $\leq_3$) is not a ${}_iT_i{}^*{}_{1/2}$ space.

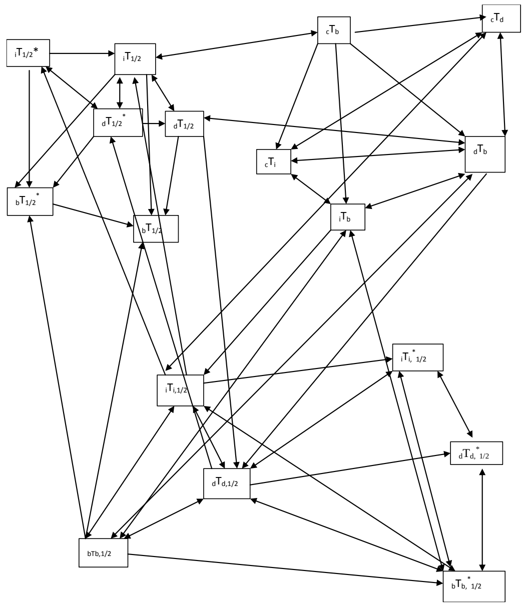
EXAMPLE 3.50. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (c , a) , (c , b)}. Clearly (X , τ_4, $\leq_3$) is a topological ordered space. ig*-closed sets are ϕ , X , {b}. i-closed sets are ϕ , X , {b}. dg-closed sets are ϕ , X , {a , b}. d-closed sets are ϕ , X . Here every ig*-closed set is an i-closed set. Therefore (X , τ_2, $\leq_3$) is ${}_iT_i{}^*{}_{,1/2}$ space. Let A = {a , b } Clearly A is an dg-closed set but not a d-closed set. Hence (X , τ_2, $\leq_3$) is not a ${}_dT_{d\ 1/2}$ space.

THEOREM 3.51. Every ${}_iT_{i,1/2}$ space is an ${}_iT_{1/2}{}^*$ space.

Proof: Let (X , τ , $\leq$) be a ${}_iT_{i,1/2}$ space . We show that (X , τ , $\leq$) is an ${}_iT_{½}{}^*$ space. Let A be an ig*-closed subset of X. We know that every g* -closed set is an g-closed set. Then A is an ig-closed subset of X. Since (X , τ , $\leq$) be an ${}_iT_{i,1/2}$ space and A is an ig-closed subset of X, then A is a closed set. Every ig*-closed set is a closed set. Then (X , τ , $\leq$) is an ${}_iT_{1/2}{}^*$ space. Thus every ${}_iT_{i,1/2}$ space is a ${}_iT_{½}{}^*$ space. The converse of the above theorem need not be true. This will be seen in the following example.

EXAMPLE 3.52. Let X = {a,b,c} , τ_4 = { ϕ , X , {a} , {a , c}} and $\leq_6$ = {(a , a) , (b , b) , (c , c) , (b , a) , (a , c) , {b , c) }. Clearly (X , τ_4, $\leq_6$) is a topological ordered space.

ig-closed sets are ϕ , X , {b , c }. closed sets are ϕ , X , {b} , {b , c} . i-closed sets are ϕ , X . Clearly every ig-closed set is a closed set. Let A = { b , c}. Clearly A is an ig-closed set but not an i-closed set. Hence (X , τ_4, $\leq_2$) is a ${}_iT_{1/2}$ space but not a ${}_iT_{i,1/2}$ space.



IV. CONCLUSION

In this paper, we will discuss the various properties between the ig*, dg* and bg* closed type sets in topological spaces.

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